Extending the Ice-Age Sea-Level Equation: Water Flux Across Sills

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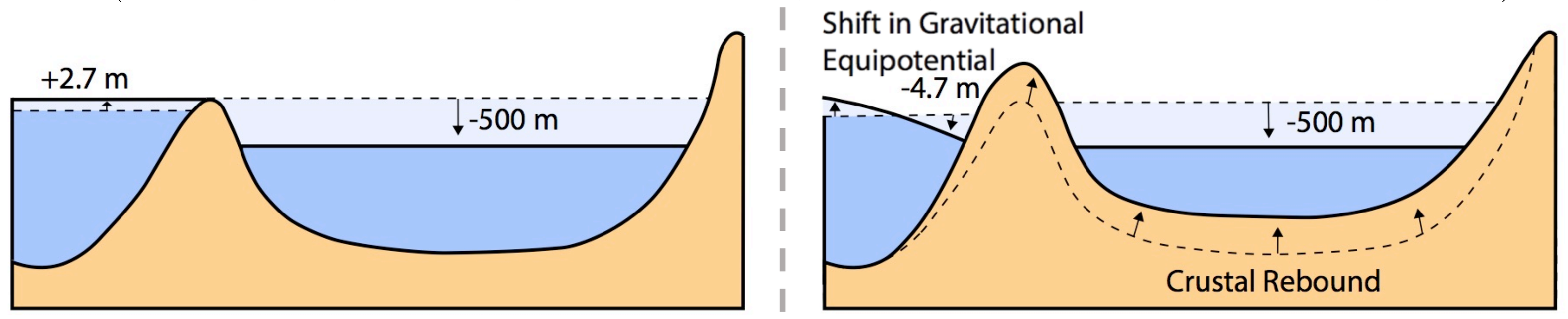
1. Introduction

▷ Previous treatments of the "sea-level equation", which governs ocean redistribution driven by ice mass flux (originally derived by Farrell and Clark, 1976), assume that any location where sea level is greater than zero – i.e., where the crust lies below the sea-surface equipotential – will experience water mass changes as ice sheets grow and diminish, even if the location is not connected to the global ocean.

▷ There are numerous cases during a glacial cycle when this assumption is not inappropriate, for example, for an isolated body of water that becomes connected to the global ocean by sill overtopping due to rising local mean sea level. Notable examples from the last glacial cycle include the isolation and reflooding of the Black Sea, the Caspian Sea, and the Persian Gulf.

▷ Similar to growth and decay of ice sheets, redistribution of water to and from these isolated water bodies also causes solid earth deformation and perturbation to the Earth's geopotential and rotation vector. We extend the generalized sea-level theory of Mitrovica and Milne (2003) to derive a gravitationally self-consistent, spatio-temporal sea-level changes within an ocean-plus-lake system that is intermittently connected by water mass flux across a sill. (For simplicity, here we present the theory for a system with no shoreline migration.)



Eustatic "Bath Tub" Scenario | Actual Predicted Change in Sea Level

2. Theory: One Water Body

The change in global sea level is defined as the difference between the change in the height of the sea-surface equipotential, G, and change in the solid surface, R.

$$SL(\theta,\psi,t) = G(\theta,\psi,t) - R(\theta,\psi,t)$$

Global sea-level change is related to ocean height changes by the ocean function, C

$$\Delta S(\theta,\psi,t) = \Delta SL(\theta,\psi,t)\cdot C(\theta,\psi)$$

The change across a single timestep (j) is then expressed as

$$\Delta S(\theta,\psi,t_j) = \Delta S(\theta,\psi,t_{j-1}) + \delta S(\theta,\psi,t_j)$$

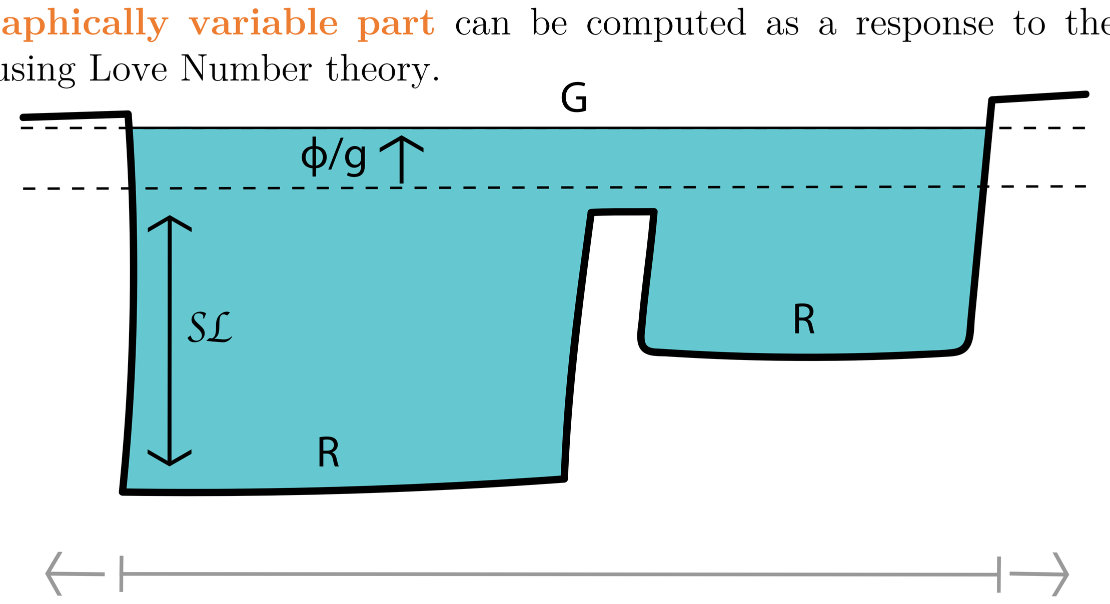
We can decompose sea-level change into a geographically variable part and a uniform shift.

$$\delta S(\theta,\psi,t_j) = \left[\delta\mathcal{SL}(\theta,\psi,t_j) + \frac{\delta\phi(t_j)}{g}\right] C(\theta,\psi)$$

The uniform shift in the height of the sea surface equipotential is calculated by invoking the conservation of mass of the surface load (i.e. the mass of ice melted is equal to the mass of water that enters the oceans).

$$\frac{\delta\phi(t_j)}{g} = -\frac{1}{A}\frac{\rho_i}{\rho_w}\iint_\Omega \delta I(\theta,\psi,t_j)\mathrm{d}\Omega - \frac{1}{A}\iint_\Omega \delta\mathcal{SL}(\theta,\psi,t_j)C(\theta,\psi)\mathrm{d}\Omega$$

The geographically variable part can be computed as a response to the total surface mass load using Love Number theory.



3. Theory: Two Water Bodies

In the ocean and lake case (denoted by o and m respectively), we have two separate sea level equations governing the redistribution of water:

$$\delta S_o(\theta,\psi,t_j) = \left[\delta\mathcal{SL}(\theta,\psi,t_j) + \frac{\delta\phi_o(t_j)}{g}\right] C_o(\theta,\psi)$$

$$\delta S_m(\theta,\psi,t_j) = \left[\delta\mathcal{SL}(\theta,\psi,t_j) + \frac{\delta\phi_m(t_j)}{g}\right] C_m(\theta,\psi)$$

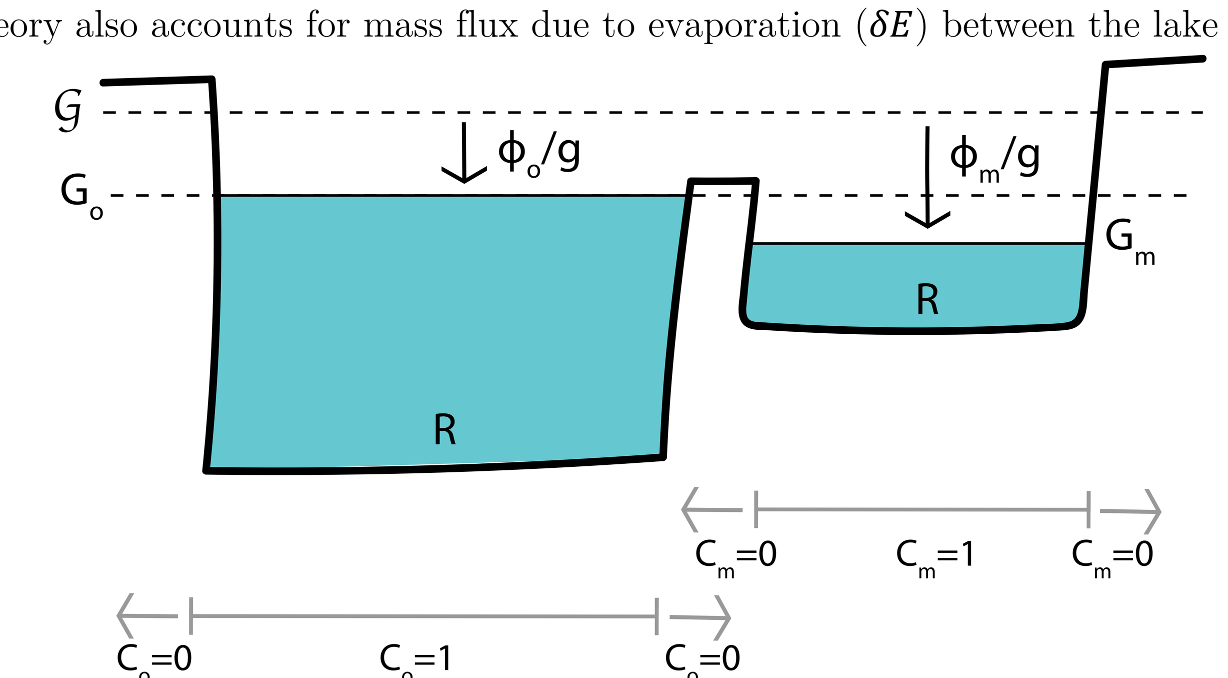
These expressions are coupled through the geographically variable component of the change in global sea level, δSL. This is because the global sea-level change is driven by the total surface mass load; so, for example, the loading history in the lake will impact sea level in the ocean, and vice versa.

When the lake is separated from the ocean it has a distinct sea-surface equipotential, which is not affected by ice melt water since the ice reservoir is only in contact with the global ocean.

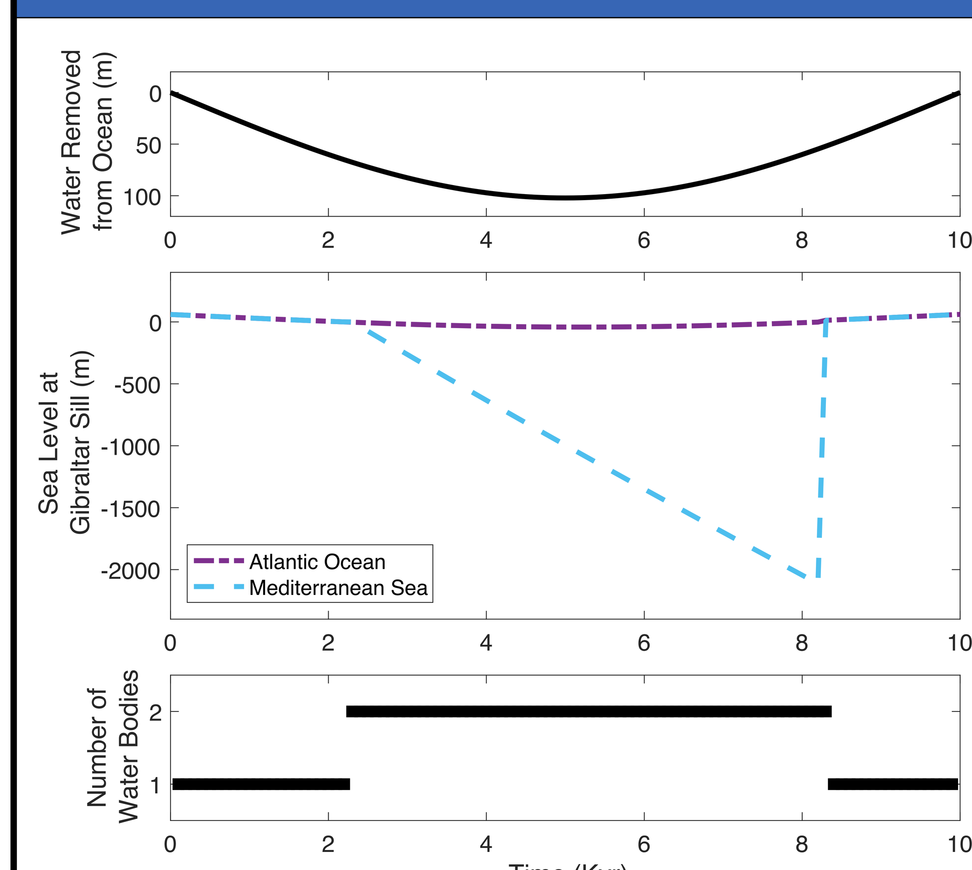
$$\frac{\delta\phi_o(t_j)}{g} = -\frac{1}{A_o}\frac{\rho_i}{\rho_w}\iint_\Omega \delta I(\theta,\psi,t_j)\mathrm{d}\Omega + \frac{\delta E(t_j)}{A_o} - \frac{1}{A_o}\iint_\Omega \delta\mathcal{SL}(\theta,\psi,t_j)C_o(\theta,\psi)\mathrm{d}\Omega$$

$$\frac{\delta\phi_m(t_j)}{g} = -\frac{\delta E(t_j)}{A_m} - \frac{1}{A_m}\iint_\Omega \delta\mathcal{SL}(\theta,\psi,t_j)C_m(\theta,\psi)\mathrm{d}\Omega$$

The theory also accounts for mass flux due to evaporation (δE) between the lake and ocean.



4. Application – Sea Level in the Mediterranean

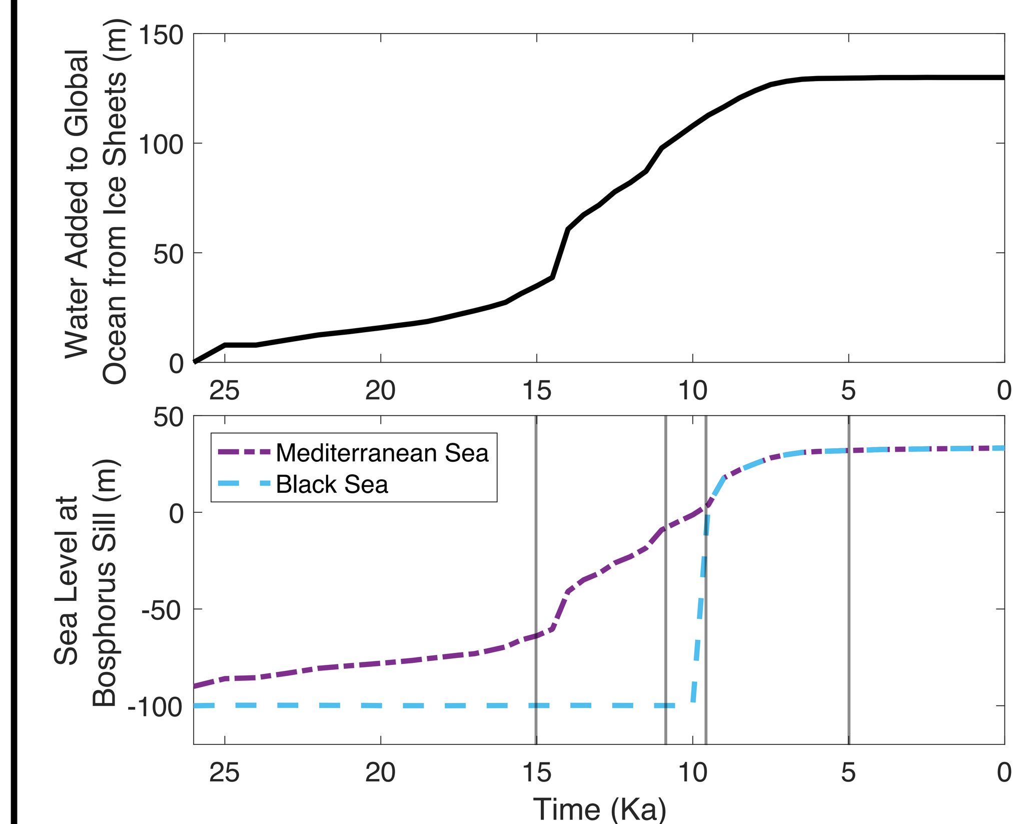


To test our new sea-level formalism, we consider a sinusoidal fluctuation in global sea level of magnitude 100 m, driven by far field ice mass changes (top panel), acting on modern day topography. We treat the Mediterranean as the lake that can become disconnected from the global ocean (Gibraltar seaway set to an initial depth of 60 m). When the Mediterranean becomes isolated from the global ocean it is no longer affected by volume flux from the ice sheets.

▷ Video showing detailed view of the evolution of topography of the region through time.

5. Application – Flooding of the Black Sea

We simulate a maximum flooding scenario for the Black Sea ~10 ka. From LGM to present, global mean sea level rose 130 m; here we adopt the ICE-6G-C ice history model (Peltier et al, 2015). The Bosphorus sill at LGM is prescribed to be 90 m above the surface of the Mediterannean Sea and 100 m above the level of the Black Sea.

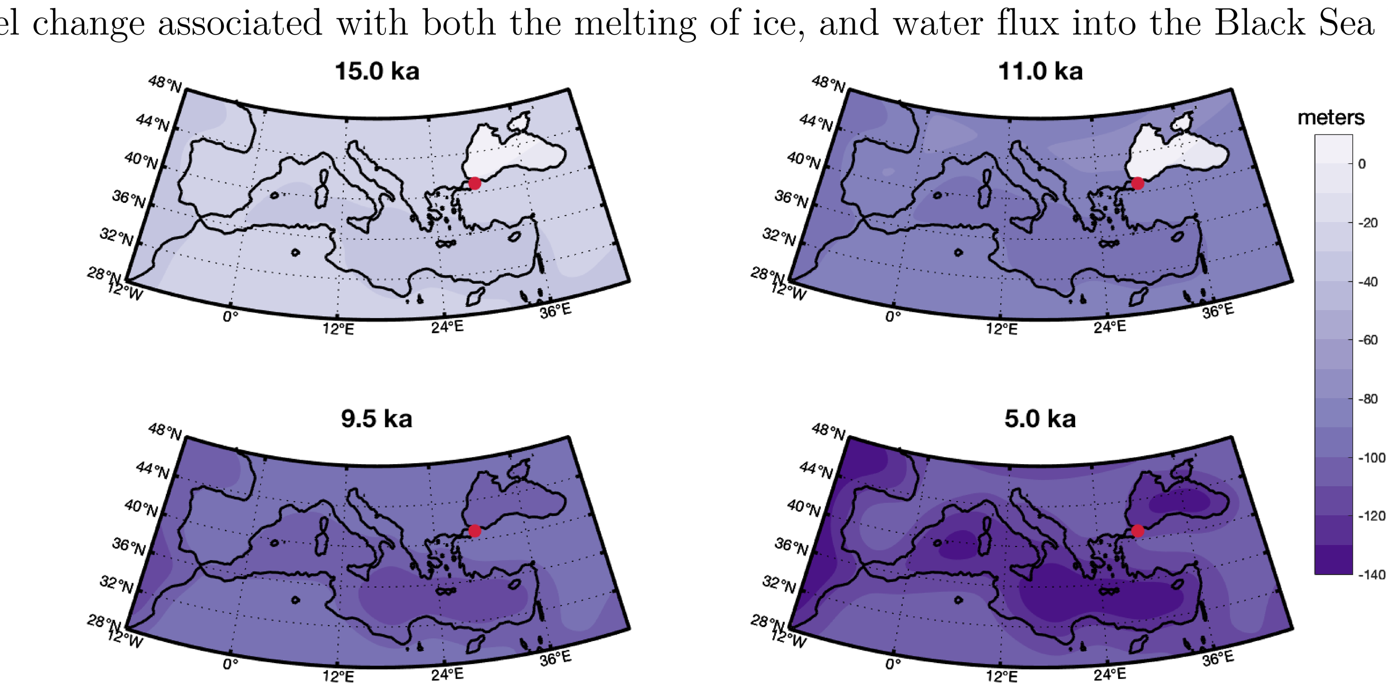


Sea-level change through time (left).

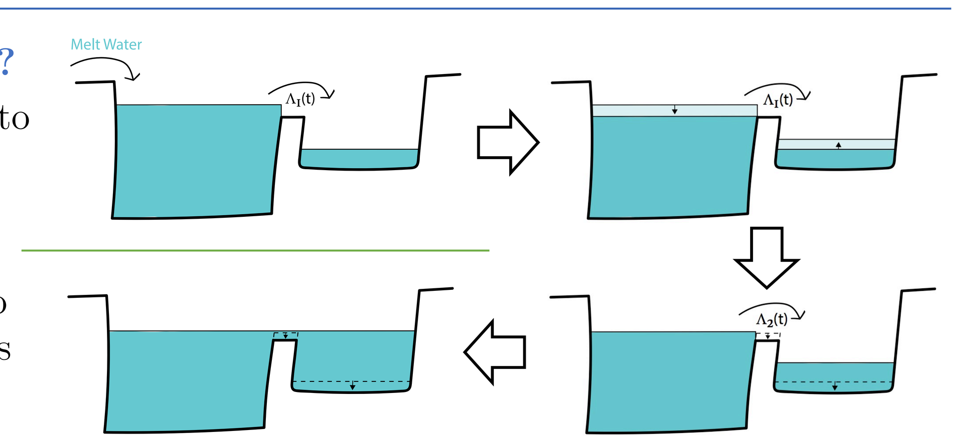
▷ The top panel shows the eustatic (or global average) height of melt water introduced into the ocean.

▷ The bottom panel gives sea level on the west side of the Bosphorus Strait, in the Mediterannean Sea, and on the east side, in the Black Sea.

Computed change in topography relative to the start of the simulation (below).
At 9.5 ka, the sea surface on the Mediterannean side of the Bosphorus sill rises above the sill height, so water begins to overtop the sill. At this stage, the simulation captures the sea-level change associated with both the melting of ice, and water flux into the Black Sea basin.



What happens during flooding?
Increased load will cause the crust to subside (at the sill), and change in gravitational potential will pull water toward the sill. This means that more water is now available to overtop the sill. This feedback leads to runaway flooding.



6. Take-Home Message

▷ We develop a new framework which allows calculation of geographically variable sea-level change for scenarios in which a lake becomes isolated from the global ocean, or is flooded from an initial lower water level, and connected to the global ocean (or any combination of the two). Our model includes a realistic treatment of time varying shorelines, changes in the perimeter of grounded, marine-based ice and Earth rotation.

▷ We illustrate the new theory with example scenarios from the Mediterranean Sea and the Black Sea flood. This demonstrates the importance of including the geophysical feedbacks associated with sea-level change in the flooding of an isolated basin.

References:
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